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Conditions for Safe Separation of External Stores

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The trajectory of external stores ejected from a carrier aircraft is studied with the goal of developing straightforward safe separation criteria. It is shown that the initial velocity and initial acceleration provide a means of defining a sufficient condition for safe separation. This condition is sufficiently simple that it can be programmed on handheld computers. Since the condition is only sufficient, any store that fails to satisfy the condition requires a further more detailed study to determine whether its separation is safe or not.

Introduction

STRIKE aircraft are required to carry an increasing variety of external stores. Each possible combination must be analyzed to insure it will separate safely. Since the number of stores combinations can increase factorially with the number of stores and racks, certifying safe separation is operationally crucial. No matter how potentially useful the store, if it damages or destroys the carrier or itself at release, it is of no practical value. For this reason the problem of defining safe store separation has received considerable attention in the last decade. Schindel[1] has recently summarized the state-of-the-art in this regard. He shows that the earlier work more or less centers on insuring the vertical clearance condition $Z-X\sin\theta>0$ for any time (see Fig. 1 for geometry). If d is the relative distance between some point r_p on the store and r_q on the aircraft, the more general form of the clearance condition is

$$d=d(0)+\int_0^t \{[R]\cdot(U+\omega x r_p)-(V_{A/C}+\Omega x r_q)\}\mathrm{d}t \quad (1)$$

Here

- $V_{A/C}$ = aircraft translational velocity in aircraft body coordinates
- Ω = aircraft rotational velocity in aircraft body coordinates
- r_q = position vector between aircraft c.g. and point of (interest) on aircraft
- r_p = position vector between origin and point on store
- $[R]$ = rotation matrix to transfer store velocities to aircraft coordinate frame
- U = store linear velocity in store coordinates
- ω = store angular velocity in store coordinates

In this notation a separation is called safe as long as $d(t)>d(0)$ for any point on the aircraft and store, and for any time.

The calculation of U and ω requires detailed information about the aerodynamic forces and moments acting in a store. This is complex, because the store is in a nonuniform flowfield, i.e., the angle of attack of the store depends upon its attitude and velocity; the streamlines are curved because of the presence of the wing, pylon and rack; the presence of the store near the wing causes an additional aerodynamic interference that must be accounted for; finally, the streamline curvature effects vary with distance from the wing.

Classically, safe separation has been determined by a trajectory analysis like that defined in Eq. (1). Both company and military analysts have accumulated large data banks, including aerodynamic characteristics for many stores alone and for the interference field of a variety of aircraft-pylon, rack combinations.[1] Some of this data resulted from wind tunnel tests where the store was supported on a separate sting, which allowed the store to be placed in a predetermined grid. Thus the static aerodynamic characteristics of the store in the aircraft's flowfield were found. In other tests the sting was controlled by a computer which computed the next point on the trajectory and moved the sting accordingly. The actual aerodynamic forces and moments acting on the store model can be fed into this computer to make the estimate of the trajectory as realistic as possible. Thus, when accurate aerodynamic, inertial and trajectory data are available, certification by trajectory analysis is accurate, although it may be expensive.

For nonswept wings the lateral force and moments acting on the store can be small. Small aberrations which are of little consequence in normal or axial motion can have an unduly large effect on lateral linear or angular velocity, or on acceleration of the store, particularly when the store is near the aircraft. However, this problem is apparently only intuitively felt, but has not been discussed in the literature (see Ref. 2, for example). For stores beneath swept wings large lateral forces and movements can exist.

Because of the similarities in geometry and inertial characteristics of various stores, many are cleared for flight by analogy. From experience, the analyst has learned when a safe separation exists and extrapolates from this.

The purpose here is to develop a simple procedure for classifying certain stores as having "Safe Separation Trajectories" using today's small programmable computers. Thus we are suggesting a quantified certification by analogy, which implies that a "safe" store is safe, while a "questionable" suggests that a more detailed analysis is needed. The point is to present a generalization of a simple sufficient condition or criterion that is applicable to a majority of cases and that can be used in conjunction with a small programmable computer to quantify the process of certification by analogy. This idea is based on earlier work[3] that has been discussed in some detail in Ref. 1. The procedure followed is to define the trajectory in terms of the initial velocity and acceleration. If these characteristics are sufficiently large in the pitch plane and small in the lateral plane, the store will have a trajectory that allows it to separate safely.

A sufficient condition for safe separation is that the store fall quickly away from the carrying aircraft. A quickly falling store will have its "safety" determined through a distance for which the first two terms are sufficient to determine the trajectory, i.e., quadratic in time. This is called a constant force approximation and seems to be reasonably accurate for stores that *will* separate safely. The need for higher order

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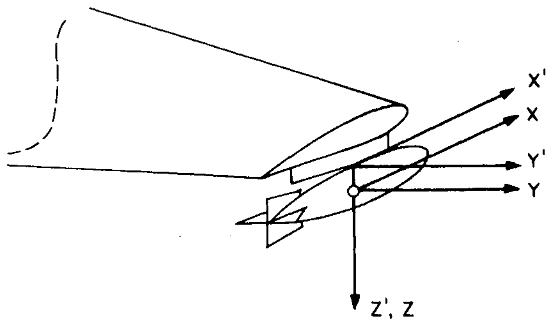

Fig. 1 Coordinate system.

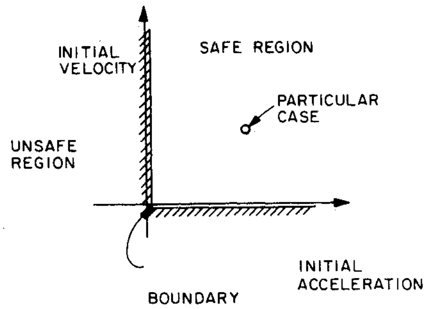

Fig. 2 Safe separation for a point mass.

terms makes the safe separation questionable and, hence, implies a need for a more accurate trajectory analysis.

The use of the constant force model will be illustrated below for several preliminary cases. Safe separation conditions will then be defined and generalized to more complex situations.

Preliminary Considerations

Consider a point mass (m), that falls vertically (Z is positive downwards) and a constant lift force (whose existence is postulated) that acts upward. Thus the equation of motion is:

$$m\frac{d^2Z}{dt^2} = (W - qSC_L) \tag{2}$$

The solution for Z is (if all other terms are constant)

$$Z = Z(0) + w_0 t + (1/2m)(W - qSC_L)t^2 \tag{3}$$

Here $Z(0)$ is the initial position and w_0 is the initial velocity. Now if $Z(t) - Z(0) > 0$; i.e., if there are no roots of Eq. (3) for any time $t > 0$, the point mass will never return to its starting point. Thus $Z(t) - Z(0) = 0$ is critical. This in turn requires that

$$t[w_0 + (1/2m)(W - qSC_L)t] = 0 \tag{4}$$

$t = 0$ is the starting condition and is of no importance. So if $w_0 > 0$ and $W - qSC_L > 0$, the point mass can never return to its initial level. This condition can be represented in a plane with an initial velocity w_0 as the ordinate and an initial acceleration $(W - qSC_L)/m$ as the abscissa. This is shown in Fig. 2. In this case the velocity-acceleration plane is divided into a safe region and a questionable region. For a given case if the coordinates for any point mass (w_0 and $W - qSC_L$) lie above and to the right of the hatched lines, the point mass will not strike the carrier.

For simplicity, normalize the velocity with respect to free fall velocity after a fall of one characteristic length and acceleration with respect to gravity. Provisionally one defines,

$$\Delta_1' = w_0/\sqrt{2gr_{\max}} \qquad \Delta_2' = 1 - [qSC_L(0)/W] \tag{5}$$

where

$$C_L(0) = C_L(Z=0) + \frac{\partial C_L}{\partial \alpha}\frac{w_0}{U}$$

The use of the term provisional here is meant to imply that the definition is incomplete.

Dynamics of Oscillatory Return

The other preliminary problem is more complicated. Its importance lies in the possibility that the ejection velocity will give rise to a vertical force that will cause the store to return to the pylon. This possibility was first considered by D.A. Jones.[4] He calculated the displacement at time $t = \pi\sqrt{I_y/-m_\alpha}$. Jones then found the roots for the constant force equation evaluated at the half period and imposed the condition of finite downward velocity at the edge of the lift interference field. The calculation given below is also based on the constant force field idea, and gives the same sort of result. But the computation is more straightforward. We assume a store that is statically stable† and of relatively low mass. The problem is to determine whether there is an ejection velocity which can cause unsafe separation. Note that in this case it is not only the distance but the rotation that is important, i.e.,

$$\delta Z = Z(t) - x\tan\theta(t) - r(x), \qquad l_t \le x \le l_N \tag{6}$$

At the store's tail r is replaced by the tail height.‡

One can solve the classic constant velocity stability equations and find, if the initial value of the angle of attack is α_0, that

$$\alpha(t) = \alpha_0 e^{-\zeta\omega_n t}\cos\omega_n\sqrt{1-\zeta^2}\,t \tag{7}$$

where

$$2\zeta\omega_n = \frac{g}{U_0}\frac{qS}{W}\left[-C_{Z_\alpha} + \frac{dU_0}{k_y 2}\left(C_{m_{\dot\theta}} + C_{m_{\dot\alpha}}\right)\right] \tag{8a}$$

$$\omega_n^2 = \frac{-qS}{W}\frac{dg}{k_y 2}C_{m_\alpha} \tag{8b}$$

To meet the needs for a sufficient condition, set $\zeta = 0$ for the remaining part of this analysis. As long as the store pitch angle remains reasonably small, use Newton's second law for the component normal to the flight path, and obtain the flight path angle, γ after one integration. The pitch angle is the sum of the angle of attack and the flight path angle. In the case of a store trajectory where Z is positive downwards and $\theta(0) = \theta_0$,

$$\theta(t) = +\alpha_0\cos\omega_n t + (g/U_0\omega_n)(qS/W)C_{Z_\alpha}\sin\omega_n t + \gamma_0 \tag{9}$$

The coefficient of the sin $\omega_n t$ term is generally small compared to unity. Hence, the oscillation has an amplitude of approximately the initial angle of attack α_0. Since the change in height is related to Z by integration of γ, one finds after

†An unstable store launched downward from beneath the wing is not included in this calculation. In fact, this calculation is not needed for unstable stores since vertical displacement lags rotation by two time integrals. Hence an unstable store is likely to hit the carrier by rotation before it is lifted up to strike the carrier aircraft.

‡One's initial inclination is to suppose the increase in drag due to rotation will cause the store to accelerate aft. This is true, but a time lag appears between the appearance of the acceleration and the actual displacement.

carrying out the integration

$$Z \simeq g\left(1+\frac{qSC_{Z0}}{W}\right)\frac{t^2}{2}+w_0\left(t+\frac{qSC_{Z_\alpha}}{W}\frac{g}{U_0\omega_n^2}(1-\cos\omega_n t)\right) \tag{10}$$

It is possible to substitute into Eq. (6) and obtain an approximate expression for the distance between the store and aircraft.

The important issue here is the relative size of the oscillatory term compared to the nonoscillatory term. When $\omega_n t=\pi$ (or $t=T/2$), the time when the oscillatory term makes the maximum negative contribution to Z, Eq. (10) takes the form

$$Z \simeq g\Delta_2\frac{\pi^2}{2\omega_n^2}+\Delta_1\sqrt{2gr_{\max}}\left(\frac{\pi}{\omega_n}-2\frac{qSC_{Z\alpha}}{W}\frac{g}{U_0\omega_n^2}\right)$$
$$-\frac{l}{2}\tan\left(-\frac{g\pi}{U_0\omega_n}\Delta_2+2\Delta_1\frac{\sqrt{2gr_{\max}}}{U_0}\right) \tag{11}$$

After normalizing Z with $r_{\max}$, the boundary ($Z/r_{\max}\geq 0$) is obtained; that is

$$\Delta_1 \geq \left[\frac{l}{2r_{\max}}\sin\left(2\frac{\sqrt{2gr_{\max}}}{U_0}\Delta_1-\frac{\pi g}{U_0\omega_n^2}\Delta_2\right)-\frac{\pi^2}{2}\frac{g}{r_{\max}\omega_n^2}\Delta_2\right]$$
$$\times\left[\sqrt{\frac{2g}{r_{\max}\omega_n^2}}\left(\pi-2\frac{qSC_{Z_\alpha}}{W}\frac{g}{U_0\omega_n}\right)\right]^{-1} \tag{12}$$

The second term in the denominator of Eq. (12) is usually negligible compared to π unless the store has almost zero static margin (a case that was excluded). The radical in the denominator represents a nondimensional characteristic time which may be written

$$2(k_y/d)\sqrt{(-W/qSC_{Z_\alpha})(-d/X_{C_p})}$$

which is fairly large.

It remains to consider the argument of the sine term, it may be written

$$2(w_0/U_0)-\pi(k_y/U_0\sqrt{g/d}\sqrt{-d/X_{C_p}}\cdot\sqrt{-W/gSC_{Z_\alpha}})\Delta_2 \tag{13}$$

The first term is twice the induced angle of attack due to downward ejection. The term in square brackets is approximately the ratio of the velocity after free fall of one diameter or less to the flight speed multiplied by the square root of the static margin. Normally this product will be small. Multiplication by π does not make the situation critical. The last term in the product is the square root of the angle of attack required for level flight, which is small for winged stores and less than one for nonwinged stores. Hence, both terms which contribute to the argument of the sine are small. So replace the sine of the angle by the angle and finally obtain

$$\Delta_1 \geq \left\{-\left(2\pi\frac{l}{U_0}\sqrt{\frac{g}{d}}\right)+\frac{\pi^2}{2}\frac{k_y}{d}\left[\left(-\frac{W}{qSC_{Z_\alpha}}\right)\left(-\frac{d}{X_{C_p}}\right)\right]^{1/2}\right.$$
$$\left.\times\left[1-4\frac{l}{k_y}\left(\frac{\rho S(-Xc_p)(-Cz_\alpha)}{2m}\right)^{1/2}\right]^{-1}\right\}\Delta_2 \tag{14}$$

Note that the denominator has been rewritten to show the second term is small, since it represents the relative mass ratio of the air to store. Equation (14) may be interpreted as defining a boundary separating the safe zone from the questionable zone in which a store may recover its initial height during the dynamic motion following release. This boundary is approximately a straight line with a negative slope through the origin. Two additional dimensionless groups appear in the slope

$$2\pi(l/U_0)\sqrt{g/d}$$

and

$$(\pi^2/2)(k_y/d)[-(W/qSC_{Z_\alpha})(-d/X_{C_p})]^{1/2}$$

The former depends solely on the store geometry and flight speed and may be rewritten in the form,

$$2\pi(l/d)(\sqrt{2gr_{\max}}/U_0)$$

It is the product of the fineness ratio and the ratio of free-fall speed to flight speed. For slender stores at very low speeds this term can approach unity. For short stores at high speeds, the term can be of the order of one one-hundredth (0.01).

The second term is about one-half times the square root of the angle of attack required for level flight. The magnitude of the second term will be approximately a few tenths to unity. However, note that the presence of these two extra terms causes this boundary to be specific to each speed and store.

It may be of interest to note that in the half period the store is convected aft a distance that is approximately

$$\frac{X}{r_{\max}} \simeq -2\pi^2\frac{C_D+\delta C_D}{-C_{M_\alpha}-\delta C_{M_\alpha}}\left(\frac{k_y}{d}\right)^2$$

where δC_D and δC_{M_α} are the interference terms.

Proposed Criteria

As in all of the previous studies, safe separation criteria here are predicted on study of the equations of motion and determination of the circumstances under which the store will not strike the carrier aircraft. In other words, the criteria are defined conditions under which the equation of the trajectory has no roots in one sense or another.

The departure used here is to use the initial velocity and acceleration (i.e., derived from approximate mate loads plus the increment the induced angle caused by ejection velocity) as predictors. We have found these velocities and accelerations must include both linear and rotational parts. Further, we assert that substantial parts of the Δ_1, Δ_2 plane must be excluded from the "safe zone."§

The first and obvious exclusion is to use only that part of the plane where

$$\text{A.} \qquad \Delta_1>0, \qquad \Delta_2>0 \tag{15}$$

David Schock (see Ref. 3) at McDonnell-Douglas has shown that the store which does not separate sharply is potentially troublesome. If under the constant force approximation we require the store to fall its maximum radius in some time t_f, we find that

$$\Delta_1=0.707(r_{\max}/t_f^2 g)^{1/2}-0.500(t_f^2 g/r_{\max})^{1/2}\Delta_2 \tag{16}$$

This equation describes a line which intersects the Δ_1 axis at $0.707\,(r_{\max}/t_f^2 g)^{1/2}$ and the Δ_2 axis at $\Delta_2=\Delta_{2f}=\sqrt{2}(r_{\max}/t_f^2 g)$. If we set $r_{\max}/t_f^2 g=1/4$, a situation in which the normal acceleration compares roughly to a 60 deg dive, then we have

$$\Delta_1=0.354-\Delta_2 \tag{17}$$

§The reader is referred to Ref. 3 for mathematical details, including the series solution which defines Δ_1 and Δ_2. Appendix A contains the formula for Δ_1, Δ_2, etc.

In this case the safe zone is

B. i) $\Delta_1 \geq 0, \qquad \Delta_2 \geq 0.354$

$$\text{ii)}\ \Delta_1 > 0.354 - \Delta_2, \qquad \Delta_2 \leq 0.354 \tag{18}$$

The final contribution to the criterion is derived from Jones' idea.[4] However, we will follow our simplified analysis with a modification that causes our result to be compatible with his. This comes about by including the effect of the initial moment acting on the store. The net effect is to redefine α_0, namely

$$\alpha_0 = \alpha_{\text{carriage}} + \frac{w_0}{U} + \frac{C_{M_0}}{C_{M_\alpha}} \tag{19a}$$

Defining

$$\lambda = \left[2\pi \frac{l}{U_0}\sqrt{\frac{g}{d}} + \frac{\pi^2}{2}\frac{k_y}{d}\left[-\frac{W}{qSC_{Z_\alpha}}\left(-\frac{d}{X_{C_p}}\right)\right]^{1/2}\right]$$
$$\times\left[1 - 4\frac{l}{k_y}\left(\frac{-\rho SC_{Z_\alpha}(-X_{C_p})}{2m}\right)^{1/2}\right]^{-1} \tag{19b}$$

we find that

$$\Delta_1 \simeq -\lambda\Delta_2 + \left(\frac{g}{2r_{\max}}\right)^{1/2}\frac{l}{\omega_N}\left[\frac{-qSC_{Z_\alpha}}{W}\ \frac{C_{M_0}}{-C_{M_\alpha}}\right]$$
$$\times\left[1 - 4\frac{l}{k_y}\left(\frac{\rho SX_{C_p}C_{Z_\alpha}}{2m}\right)^{1/2}\right]^{-1} \tag{20a}$$

For convenience, let

$$\eta = \left[\left(\frac{g}{2r_{\max}}\right)^{1/2}\frac{l}{\omega_N}\left(\frac{-qSC_{Z_\alpha}}{W} + \frac{C_{M_0}}{-C_{M_\alpha}}\right)\right]$$
$$\times\left[1 - 4\frac{l}{k_y}\left(\frac{\rho SX_{C_p}C_{Z_\alpha}}{2m}\right)^{1/2}\right]^{-1} \tag{20b}$$

Now if $\lambda < 1$, our form of Jones' criterion lies below Schock's "minimum-time-to-fall line." On the other hand, if $\lambda > 1$, then the Jones' line will intersect the Schock line at

$$\Delta_{1J} = 0.5\lambda - \eta/(\lambda - 1) \tag{21}$$

and

$$\lambda_{2J} = -0.5 - \eta/(\lambda - 1) \tag{22}$$

Hence, for safety, one also requires the calculated value of Δ_1 and Δ_2

$$\Delta_1 > \Delta_{1J}$$

and

$$\Delta_2 > \Delta_{2J} \tag{23}$$

Briefly, it seems possible to define in the Δ_1, Δ_2 plane a sufficient condition that delineates those stores that will separate safely from those which may not. The sufficient condition includes the boundary in the Δ_1, Δ_2 plane [Eqs. (18)] and the nonreturn condition [Eq. (23)].

Figure 4 shows this vertical plane. Hence high ejection velocities provide safe clearance for the store in the left half plane (i.e., negative initial acceleration). This is also applicable to winged stores of low drag, which is why one calculates Δ_{1J} and Δ_{2J} if $\lambda > 1$.

In practice then, one computes Δ_1 and Δ_2 for the nose, the highest point on the fin and the wing tips, if there is a wing. Then if $\lambda < 1$, use Eqs. (18) solely. If $\lambda > 1$, Eqs. (18) apply together with Eq. (23). When these conditions are satisfied, the separation is said to be safe.

One can use the diagram (Fig. 3) to trace out a locus for different flight conditions and to determine the circumstance under which a store may be headed for trouble. This is illustrated crudely in Fig. 4. In the actual calculation increasing "q" reduces Δ_2 but the balance between C_Z and C_M may shift because of the change in Mach number. Hence, it is not obvious how the locus will vary in detail.

The lateral condition is the same as that given in Ref. 3 (or Appendix B), that is, the store must fall below the adjacent solid boundary before the store moves laterally the distance to that boundary.

The details of the computation are given in Appendix A for steady flight. There the effects of maneuvering flight in the various Δ's are outlined.

Example

There is a paucity of published data to evaluate the criteria described above. Maddox's[5] recent analysis of store trajectories provides information which can be used for this purpose. The appropriate aerodynamic data can be found in Ref. 2 of Maddox's paper and are tabulated in Table 1.

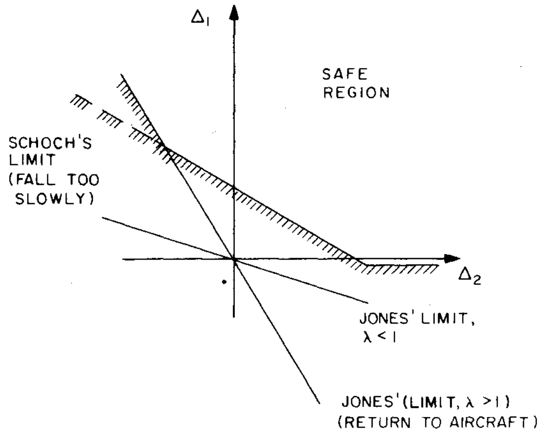

Fig. 3 Safe separation region in pitch (Δ_1, Δ_2 plane).

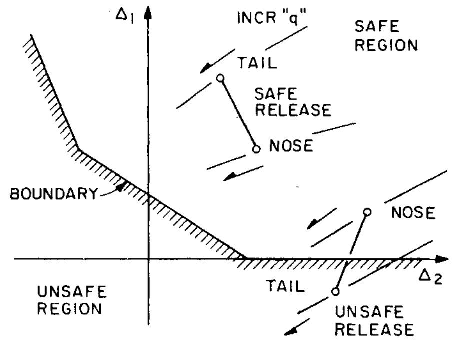

Fig. 4 Typical safe separation boundary/vertical plane for two specific stores.

Table 1 Aerodynamic data

M	C_N $(-C_Z)$	C_Y	C_m	C_n	C_l	qS/W
0.6	0.15	−0.35	−1.0	0.20	0.10	0.471
0.8	0.05	−0.20	−1.3	0.05	0.08	0.831
0.9	0.02	−0.10	−1.4	0.05	0.08	1.052

Table 2 Store separation parameters

M	Δ_1	$\Delta_{2_{tail}}$	$\Delta_{2_{nose}}$	Δ_{1J}	Δ_{2J}	Δ_3	$\Delta^a_{4_{tail}}$	A_{tail}	A_{nose}
0.6	0.68	0.188	0.594	0.517	−0.017	0	−1.230	0.22	1.30
0.8	0.70	−0.741	0.129	0.544	−0.044	0	−0.612	b	1.08
0.9	0.63	−1.338	−0.169	0.557	−0.056	0	−0.637	b	0.80

[a]To allow for fin rotation, Δ_4 is redefined for straight and level flight as

$$\Delta_4 = \frac{qS}{W}\left(C_Y + \frac{xd}{k_z^2}C_n - \frac{h_T d}{k_x^2}C_l \sin\phi_F\right)$$

The added term in C_l allows for lateral motion of the fin due to roll. A similar term could be added to Δ_2 to allow for vertical motion. This term is $+(h_T d/k_x^2)C_l \times \cos\phi_F$. The initial fin angle ϕ_F, was taken to be 45 deg.

[b]A is complex for these conditions.

The parameters Δ_1, etc., were calculated assuming standard conditions at the listed altitude, the center of mass was approximately one-third the length of the store aft the nose, and the value of slope of the normal force and pitching moment coefficient curves could be estimated using standard techniques. The results are given in Table 2.

These results show that the nose of the store is not likely to present problems. However at $M=0.8$ and 0.9 the tail could present problems both vertically and laterally. The nose down pitching moment is the dominant factor in calculating Δ_2. The complex value for A implies that the tail falls very slowly, if it falls at all. The lateral problem is thus the more severe principally because the roll causes the fin tip to move toward the adjacent store.

The safe separation criterion shows that the store separation is safe at $M=0.6$ but that a more detailed analysis is needed at $M=0.8$ and 0.9. Maddox's data show that the separation is safe at $M=0.6$ and 0.8 and that there is a fin strike at $M=0.9$.

A more detailed examination of Maddox's Figs. 3-5 confirms the strong nose down pitching trend which increases with increasing Mach number, as implied by the fact $\Delta_{2_{nose}} > \Delta_{2_{tail}}$. There is also a tendency for the tail to swing to the port ($\Delta_4 < 0$). Since A is related to the reciprocal of the time to fall a fixed distance, the reduction of A_{nose} with increasing Mach number is also constant with the trends in the data.

Concluding Remarks

In view of the well-known variation in aerodynamic interference terms near a combination of the rack, pylon, wing or fuselage, one may well wonder about the validity of the constant force approximation. This approximation surely idealizes the actual initial conditions as well. Consider the relative time scales in a safe separation process. The ejection system completes its action in a few milliseconds. Hence the correction of the carriage loads for this increment in angle of attack or side slip provides a reasonable estimate to the generalized forces acting on the store initially. Further, a store which separates safely and clearly is some substantial fraction of a chord length below the rack in hundreds of milliseconds.

Objections have been raised on the grounds that near the wing, the flow follows the wing so the initial angle of attack is small, but as the store moves away from the wing it encounters the full angle of attack. This is true. However, the argument is not properly quantified. Except near the leading edge of the wing, the change in flow angle is relatively slow. Rough calculations suggest the flow angle is about 15% of freestream angle of attack a quarter chord below most of the airfoil, and the same value is about 25% a half chord below the airfoil. For large distances the flow angle is roughly (α freestream) × [1 − 8/(number of chord lengths)]. Thus in the crucial near field, the relative change in the flow angle is not large.

If the store separation is marginal and the store fails to depart smartly, one must include all the details of the trajectory including variation of aerodynamic interference with distance along the trajectory. Then, the criteria based on the constant force approximation will only suggest a more detailed analysis is needed.

The determination of the constants in the criteria is empirical. If desired they could be defined to be less conservative, if adequate data were available.

Appendix A: Summary and Computation Outline

The purpose of these appendices is to describe the procedure of computing the several $\Delta_1, \Delta_2, \Delta_3$, and Δ_4 so that the analyst can determine whether or not the store will separate safely.

The procedures described here are based upon the following assumptions: a) steady aircraft motion, b) the store is rigidly connected to the airframe, and c) the airframe is rigid.

In this case the relative velocity between the store and the reference plane is zero at a time just before the start of the release sequence. With this simplification

$$\Delta_1 = [-\Delta\omega_y x(\cos\psi_0 \sin\theta_0 \sin\theta_0' + \cos\theta_0\cos\theta_0') + \Delta\omega_x z(\cos\psi_0\cos\theta_0\sin\theta_0' + \sin\theta_0\cos\theta_0') + w_0 + \Delta\omega_x y(\cos\theta_0 - \cos\theta_0')]/\sqrt{2gr_{max}} \qquad \text{(A1)}$$

Here w_0 is the ejection velocity and $\Delta\omega_y$ is the difference between the angular velocity imparted to the store by the ejector system and the angular velocity of the aircraft.

If assumption a is not satisfied, Δ_1 is defined in Eqs. (A12) and (A13). In both cases the value for z is $-r(x)$, where $r(x)$ is the radius of the store at position x.

$$\Delta_2 = \Big[\cos\Phi_0'\cos\Theta_0' + \frac{1}{2}\rho U^2(0)\frac{S}{W}\Big((C_{Z_0}+\delta C_{Z_0}) - \frac{dx}{k_y^2}(C_{M_0}+\delta C_{M_0}) + \frac{by}{k_x^2}(C_{l_0}+\delta C_l)\Big) \times \Big(\cos\psi_0\sin\Theta_0\sin\Theta_0' + \cos\Theta_0\cos\Theta_0'\Big)\Big] + \Big[\frac{1}{2}\rho U^2(0)\frac{S}{W} \times \Big((C_{X_0}+\delta C_{X_0}) + \frac{dr(x)}{k_y^2}(C_{M_0}+\delta C_{M_0})\Big) \times \Big(\cos\psi_0\sin\Theta_0'\cos\Theta_0 - \sin\Theta_0\cos\Theta_0'\Big)\Big] + \frac{1}{2}\rho U^2(0)\frac{S}{W}\Big((C_{Y_0}+\delta C_{Y_0}) + \frac{dx}{k_y^2} \times (C_{N_0}+\delta C_{N_0})\Big)\sin\psi_0\sin\Theta_0' \qquad \text{(A2)¶}$$

If the aircraft is pitching, etc., C_{M_0} includes the pitch rate

¶Note $y=0$ for axisymmetric stores.

effect, and the

aircraft roll angle at release $=\Phi_0'$
aircraft pitch angle at release $=\Theta_0'$
aircraft yaw angle at release $=\Psi_0'$

Lateral Motion (see Appendix B)

$$\Delta_3 = [\,(\Delta\omega_x z - \Delta\omega_y x)\sin\psi_0 \sin\Theta_0 + v_0\,]/\sqrt{2gr_{\max}}$$

$$\Delta_4 = \cos\Theta_0 \sin\phi_0 + \frac{qS}{W}\left[\left(C_{y_0} + \delta C_y + \frac{xb}{k_z^2}(C_{N_0} + \delta C_N)\right)\right]$$
$$\times\cos\psi_0 + \frac{yb}{k_x^2}(C_{l_0} + \delta C_l)\sin\phi_0 \cos\Phi_0$$
$$+\left(C_Z + \delta C_{z_0} - \frac{xd}{k_y^2}(C_{M_0} + \delta C_M)\right)\sin\phi_0\cos\Theta_0\Big] \qquad \text{(A3)}$$

The several terms have been defined previously. If

$$\psi_0 = 0, \qquad \phi_0 = 0$$

$$\Delta_4 \simeq \left[\sin\phi_0' + \frac{1}{2}\rho U^2(0)\,\frac{S}{W}\left(C_{y_0} + \delta C_{y_0}\right.\right.$$
$$\left.\left. + \frac{dx}{k_y^2}(C_{N_0} + \delta C_{N_0})\right)\right] \qquad \text{(A4)}$$

Axial Motion

$$\Delta_5 = (-\Delta\omega_y x \sin\Theta_0 - \Delta\omega_z x \sin\psi_0)/\sqrt{2gr_{\max}} \qquad \text{(A5)}$$

$$\Delta_6 = \left[-\frac{1}{2}\rho U^2(0)\,\frac{S}{W}\left((C_{x_0} + \delta C_{x_0}) - \frac{dx}{k_y^2}(C_{m_0} + \delta C_{m_0})\right)\right.$$
$$\times\left(\cos\psi_0 \sin\Theta_0 \sin\Theta_0' + \cos\Theta_0\cos\Theta_0'\right)$$
$$+\frac{1}{2}\rho U^2(0)\,\frac{S}{W}\left((C_{z_0} + \delta C_{z_0}) + \frac{dr(X)}{k_y^2}(C_{m_0} + \delta C_{m_0})\right)$$
$$\left.\times\left(\cos\psi_0 \sin\Theta_0' \cos\Theta_0 - \cos\Theta_0' \sin\Theta_0\right) - \sin\Theta_0'\cos\phi_0'\right] \qquad \text{(A6)}$$

Consequences of Accelerated Motion

To illustrate the effects of accelerated motion, consider an airplane moving along the arc of a circle at constant angle of attack. Hence, $\gamma = \theta$. $\gamma > 0$ implies that the airplane is flying along a concave upward flight path, otherwise $\gamma < 0$. The greater than zero case corresponds to plus "g" and the contrary corresponds to minus "g." While flying in an arc, the aircraft center of mass rises a distance, h, where

$$h = \frac{U_0^2}{g(n-1)}\left\{1 - \cos[\gamma(t) - \gamma(0)]\right\} \qquad \text{(A7)}$$

To account for this change in altitude we must express h in the rack coordinates, and then expand h in a series form. The net result gives

$$\delta z' = \frac{1}{2}gt_c^2\left[\Delta_2 + n\cos(\theta_0 - \gamma_0) - \cos\theta_2\cos\Phi_0\right]$$
$$+ t_c\left[\Delta_1\left(\frac{2r_{\max}}{g}\right)^{1/2} - x_{cg}\,\frac{(n-1)}{V_0}\,g\right] \qquad \text{(A8)}$$

Hence accelerated flight gives rise to additional terms that must be added to Δ_2 and Δ_1, defined in Eq. (A1) and (A2) as shown in Eq. (A8).

Note that $n = 1$ implies unaccelerated flight

$$n \gtrless 1 \quad \text{implies } \pm g \qquad \text{(A9)}$$

The effect of normal acceleration on the store trajectory is twofold. First, the relative velocity between the store and the rack can be altered by the rotation rate of the aircraft. In other words, the aircraft rotates about its center of mass. A store-fixed observer located away from the aircraft center of mass detects the apparent linear velocity of the rack. This apparent linear velocity is the product of the angular velocity of the aircraft and the distance between the rack reference point and the aircraft center of mass. When the aircraft center of mass is ahead of the store, the rotation tends to reduce the relative vertical velocity between the store and the aircraft. Under normal circumstances this relative velocity is small. Second, the relative velocity between the store and the aircraft can be increased by a pull-up maneuver.

Finally, consider the effect of roll rate ($\delta\omega_x$) in body axes. Roll in a vertical velocity of $\delta\omega_x y_R \cos\eta$ if η is the inclination of the principal longitudinal axis of the aircraft with respect to the rack. y_R is the spanwise portion of the rack. A positive roll rate of $\delta\omega_x$ rad/s reduces the value of the relative velocity on the starboard wing, $y_R > 0$, and increases the relative velocity on the port wing. Hence an additional increment is added to Δ_1 [Eq. (A1)]. Thus one used Eq. (A9) to compute the store position on the Δ_1, Δ_2 plane after accounting for both relative velocities due to maneuvering.

$$\Delta_1 + \left(-\delta\omega_x y_R - \frac{x_{cg}g}{V_\infty}(N-1)\right)/\sqrt{2gr_{\max}} \qquad \text{(A10)}$$

where

$$N = n\cos(\theta_0 - \gamma_0) - \cos\theta_0\cos\phi_0$$

The following information is needed to complete the calculations indicated in Eqs. (A1-A9).

Geometric

a) Store: radius distribution, including fins, or wings, if any; length; and location of the store center of mass.

b) Rack: foot position with respect to store center of mass; and location of nearest strike point in each direction.

c) Euler angles of store and rack at launch.

Force and Mass Properties

a) Store: weight and moment of inertia about each axis.

b) Rack: the rack influences the initial conditions of the store. The initial velocity is related to the ejector foot force vs distance or time for each rack load condition. The easiest calculation for velocity is to use the force distance relation and store mass.

c) Aircraft flight speed and dynamic pressure.

Store Aerodynamic Coefficients in Suspended Position at Aircraft Maneuver Conditions, Velocity and Mach Number

If the details (for example, the rack ejector impulse) are not known exactly, the best possible approximation should be used. If possible, this approximation should underestimate the velocity imparted by the ejection system to the store.

The procedure for this calculation is as follows: Compute first the angular transformations [Eq. (A1), for example]. For the rack and store then compute $v, w, u, \Delta\omega_u, \Delta\omega_z$ from the rack characteristics. Using x and $r(x)$ gives Δ_1, Δ_3 and Δ_5. Use of the nose and tail length l_N, $r=0$ and $-l_T$ and tail semiheight is usually sufficient. The aerodynamic coefficients and the aerodynamic interference terms should now be computed. The total air loads are added to the gravitational factor to give Δ_2 and Δ_4 (and Δ_6).

Finally, if the store is statically stable, one computes

$$\lambda=\left[2\pi\frac{l}{V_0}\sqrt{\frac{g}{d}}+\frac{\pi^2}{2}\frac{k_y}{d}\left(\frac{W}{qSC_{Z_\alpha}}-\frac{x_{cp}}{d}\right)^{1/2}\right]\times\left[1-4\frac{l}{k_y}\left(\frac{\rho Sx_{cp}C_{Z_\alpha}}{2m}\right)^{1/2}\right]^{-1}$$

and

$$\eta=\left(\frac{g}{2r_{\max}}\right)^{1/2}\frac{l}{\omega_n}\left(\frac{qSCz_\alpha}{W}+\frac{C_{M_0}}{-C_{M_\alpha}}\right)\times\left[1-4\frac{l}{k_y}\frac{(\rho Sx_{cp}C_{Z_\alpha})^{1/2}}{2m}\right]$$

and

$$\Delta_{1J}=\frac{0.354\lambda-\eta}{\lambda-1}$$

$$\Delta_{2J}=-\frac{0.354-\eta}{\lambda-1}$$

Safe separation requires the computed values:

$$\Delta_1\geq 0 \quad \text{if} \quad \Delta_2>0.354$$

or

$$\Delta_1\geq 0.354-\Delta_2 \quad \text{if} \quad \Delta_2<0.354$$

$$\Delta_2>\Delta_{2J}$$

$$\Delta_1>\Delta_{1J}$$

In the lateral case, from Eq. (B4)

$$\frac{\delta y'-}{2r_{\max}}A-\frac{\Delta_4}{2A}\leq\Delta_3\leq\frac{\delta y'+}{2r_{\max}}A-\frac{\Delta_4}{2A} \qquad \text{(A11)}$$

The safe separation region is a strip bounded by two slanted lines. The value of Δ_3 must lie as defined in Eq. (A-11). The slope and intercept are defined by constant force approximation; i.e., the mate loads. The intercept on the Δ_3 axis is determined by $\delta y'$ (0). The larger the distance $\delta y'(0)$, the bigger the safe domain. For many practical calculations $\delta z'$ can be taken to be one diameter or one fin span. This boundary is fixed by a particular store aircraft configuration.

Appendix B: Lateral Criteria

Let us consider the lateral motion. Here we have alternatives. The first, simplest and most conservative alternative is simply to require the store move away from the nearest store.

$$\Delta_3/\Delta_4>0 \qquad \text{(B1)}$$

In this case, depending upon the direction of the nearest store, either the first or third quadrants are acceptable, as sketched in Eq. (B1).

Note that the first quadrant applies if the store must move to the left ($\delta y'(1)<0$) to strike something solid, while the third quadrant applies to the case where movement to the right is dangerous.

However, there is a better alternative available to define a boundary in this plane. This boundary comes about because the lateral motion of a store in some cases may be limited. For safety we will require that if a store moves a distance $\delta y'$ (the lateral distance to the object it may strike), then it must also fall a distance $\delta z'$ in the *same* time ($\delta z'$ is the distance the store must fall to pass under the object at $\delta y'$). Arguing as above,

$$\Delta_3=0.707\frac{\delta y'}{r_{\max}}\left(\frac{r_{\max}}{t^2g}\right)^{1/2}-0.354\left(\frac{t^2g}{r_{\max}}\right)^{1/2}\Delta_4 \qquad \text{(B2)}$$

Now t, the time to fall the distance $\delta z'$, is for the constant force law

$$t=\frac{-\Delta_1+(\Delta_1^2+2\delta Z'\Delta_2)^{1/2}}{\Delta_2} \qquad \text{(B3)}$$

Substituting and simplifying

$$\Delta_3=\frac{\delta y'}{2r_{\max}}A-\frac{\Delta_4}{2A} \qquad \text{(B4)}$$

where

$$A=\frac{\Delta_2/\Delta_1}{\left(1+\frac{2\delta Z'}{r_{\max}}\frac{\Delta_2}{\Delta_1^2}\right)^{1/2}-1} \qquad \text{(B5)}$$

Note: if $\Delta_1\to 0$, $A\to 1$.

This line in the Δ_3,Δ_4 plane has a negative slope and limits the useful area. Thus the safe separation region in the lateral plane is determined by two slanted lines. The intercept on the Δ_3 axis is determined by $\delta y'(0)$. The larger the distance $\delta y'(0)$, the bigger the useful area. For many practical calculations $\delta z'$ can be taken to be one diameter or one fin span. Note that this boundary is fixed by each particular store-aircraft situation.

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